CONSTRAINED INDEPENDENT VECTOR ANALYSIS WITH REFERENCES: ALGORITHMS AND PERFORMANCE EVALUATION

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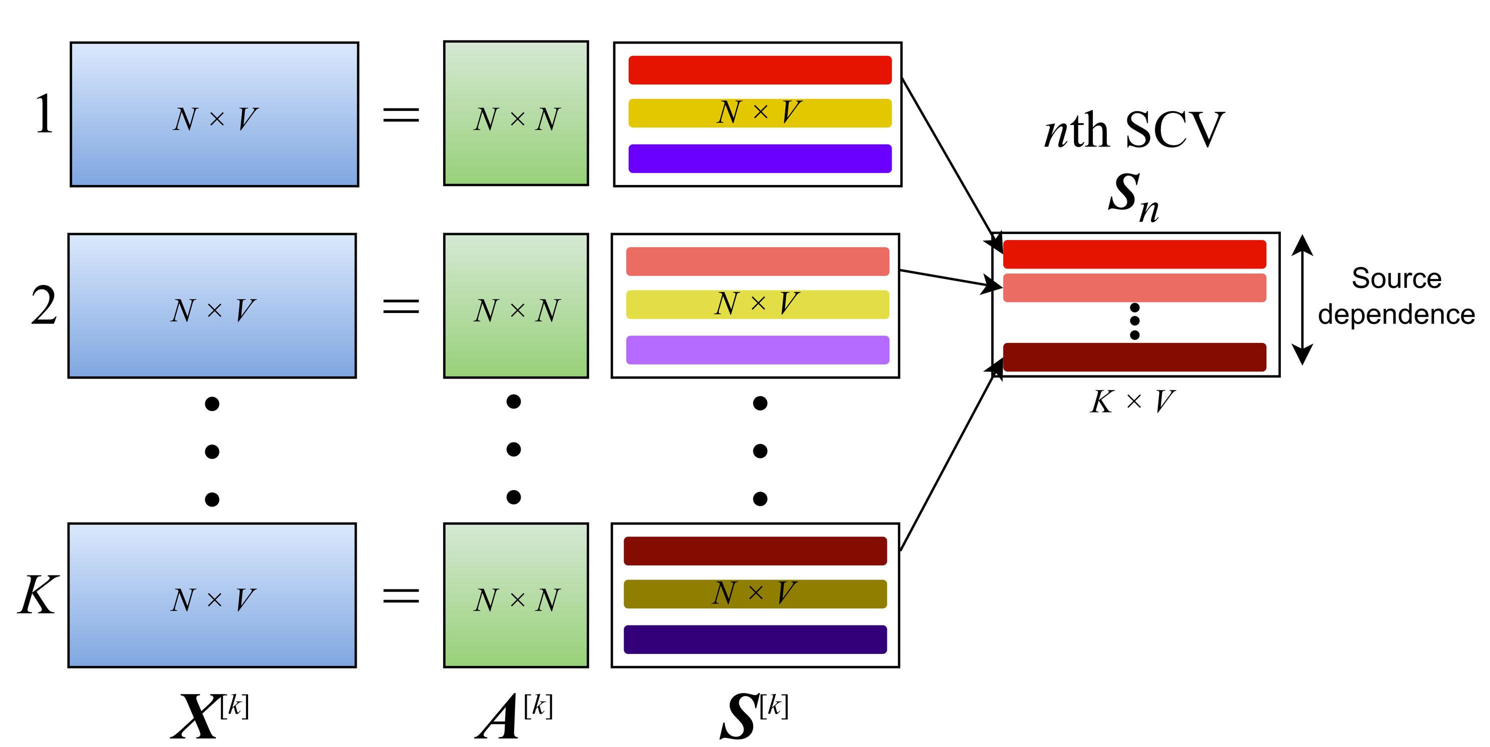
Introduction

- Joint blind source separation (JBSS) has been applied to various **neuroimaging** domains including multi-subject fMRI data analysis
- Independent vector analysis (IVA) is a powerful approach to JBSS that exploits the **statistical dependencies across datasets**
- However, IVA performance degrades when the number of datasets increases or when the level of variability among the subjects is low
- Constrained IVA (cIVA) is an effective way to bypass computational issues of IVA and improve the quality of separation by incorporating available **prior information**

Contributions

- Develop different optimization methods for cIVA
- Show their superior performance compared with IVA in different settings of constraints
- Demonstrate cIVA algorithms provide meaningful and interpretable results from analyzing real fMRI data

Independent Vector Analysis (IVA)

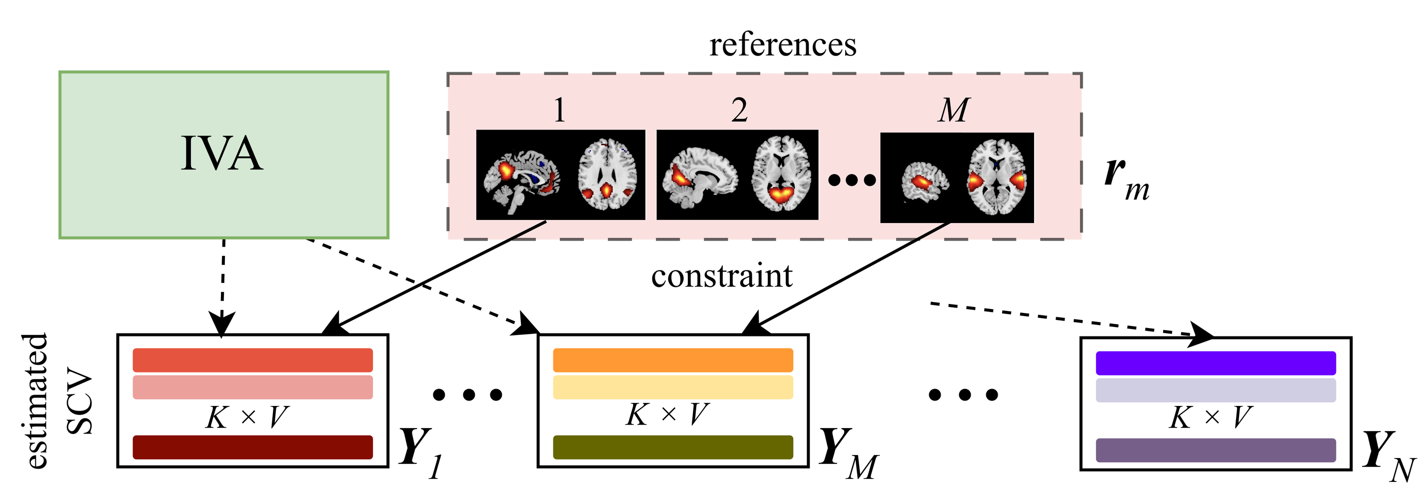


- The IVA cost function

$$\mathcal{J}_{\text{IVA}}(\boldsymbol{W}) \triangleq \sum_{n=1}^{N}\Big(\sum_{k=1}^{K}\mathcal{H}(y_n^{[k]}) - \mathcal{I}(\boldsymbol{y}_n)\Big) - \sum_{k=1}^{K}\log\left|\det(\boldsymbol{W}^{[k]})\right|$$

where $\mathcal{H}(y_n^{[k]})$ is the entropy of the nth estimated source for the kth dataset, $\mathcal{I}(\boldsymbol{y}_n)$ is the mutual information of the nth estimated source component vector (SCV), and $\boldsymbol{W}^{[k]}$ is the demixing matrix for the kth dataset.

Constrained IVA (cIVA)



- Constrained formulation of IVA with M references ($M \leq N$)

$$\min_{\boldsymbol{W}} \mathcal{J}_{\text{IVA}}(\boldsymbol{W}) \text{ s.t. } \epsilon(\boldsymbol{r}_m, \boldsymbol{y}_m^{[k]}) \geq \rho_m^{[k]} \quad \forall m = 1, \dots, M \text{ and } k = 1, \dots, K$$

Algorithms for Constrained IVA

Augmented Lagrangian (AL)

$$\mathcal{L}_{\gamma,\boldsymbol{\rho}}(\boldsymbol{W}, \boldsymbol{\mu}) = \mathcal{J}_{\text{IVA}}(\boldsymbol{W}) + \tfrac{1}{2\gamma}\textstyle\sum_{m,k}\left(\left(\max\left(0, \mu_m^{[k]} + \gamma\left(\rho_m^{[k]} - \epsilon(\boldsymbol{r}_m, \boldsymbol{y}_m^{[k]})\right)\right)\right)^2 - (\mu_m^{[k]})^2\right)$$

- AL includes **a penalty term** to the IVA cost to enforce the constraint and **a Lagrange multiplier term** to avoid numerical instabilities
- Demixing vectors can be updated via Newton direction

Alternating Direction Method of Multipliers (ADMM)

$$\min_{\boldsymbol{W},\boldsymbol{Z}} \mathcal{J}_{\text{IVA}}(\boldsymbol{W}) + \mathbb{I}_{\mathcal{C}}(\boldsymbol{Z}) \qquad \text{s.t. } \mathcal{A}(\boldsymbol{W}) - \boldsymbol{Z} = \boldsymbol{0}$$

- ADMM blends the **decomposability** of dual ascent with **strong convergence properties** of AL

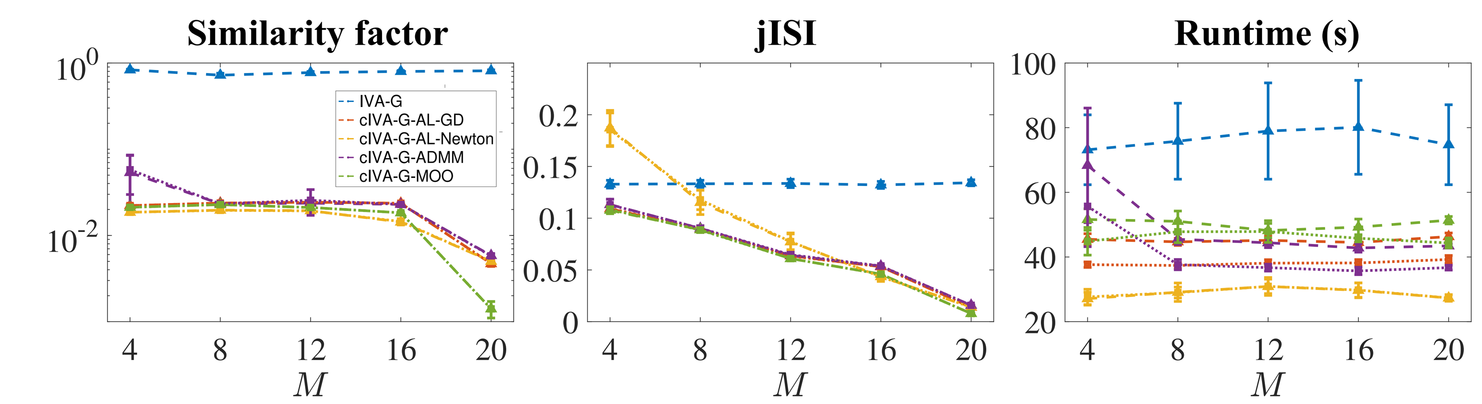
Multi-Objective Optimization (MOO)

$$\min_{\boldsymbol{W}} \mathcal{J}_{\text{IVA}}(\boldsymbol{W}) + \frac{\lambda}{2}\sum_{m=1}^{M}\sum_{k=1}^{K}\Big(\sum_{\substack{n=1\\n\neq m}}^{M}\epsilon^2(\boldsymbol{r}_m, \boldsymbol{y}_n^{[k]}) - \epsilon^2(\boldsymbol{r}_m, \boldsymbol{y}_m^{[k]})\Big)$$

- MOO adds a *regularization* cost to
  - **maximize** the similarity between the reference and the **corresponding** source component
  - **minimize** the similarity between the reference and **other** estimated source components

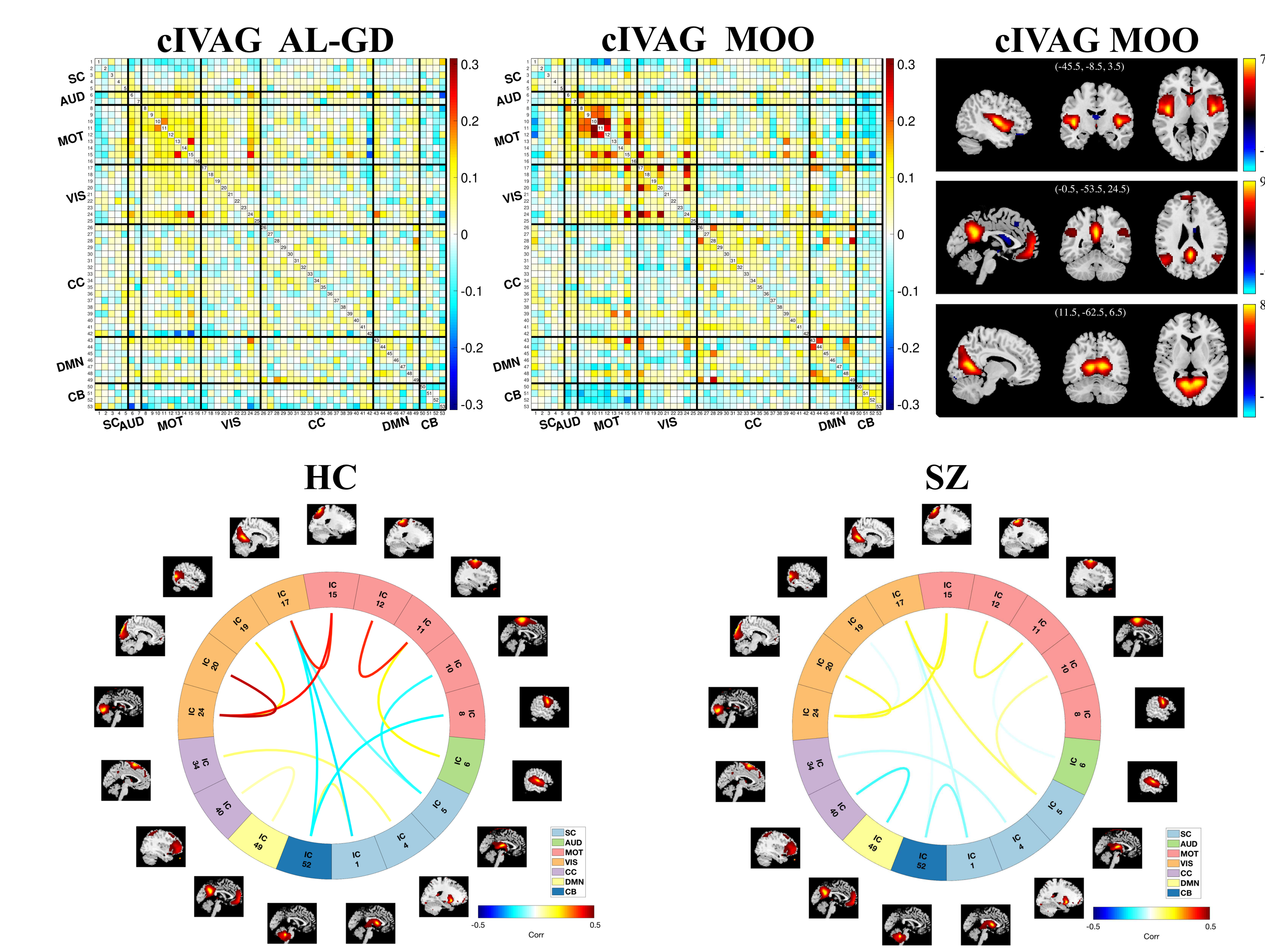
Experimental Results

Hybrid simulation — Varying number of references



- cIVA algorithms remarkably outperform (unconstrained) IVA
- MOO slightly outperforms other cIVA algorithms but is more computationally expensive

fMRI data analysis — $K = 98$ subjects



Summary

- MOO shows more meaningful and interpretable results when applied to real fMRI data
- MOO preserves subject variability and shows significant group differences between healthy control (HC) and schizophrenia patients (SZ)

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